1 Mini-courses

Nathan Dunfield : Practical computation with hyperbolic 3-manifolds

Thurston-Perelman Geometrization shows that all closed 3-manifolds can be canonically divided into pieces with geometric structures. From this, it follows that there is an algorithm for solving the homeomorphism problem for such manifolds. The most common geometry, by far, is hyperbolic geometry. While some hyperbolic 3-manifolds arise from number-theoretic constructions, most do not, though they still have associated number fields and quaternion algebras.

These lectures will outline the theory behind various numerical and algebraic methods for studying such hyperbolic structures in practice. The lectures will feature demonstrations of the SnapPy program (`https://snappy.computop.org`) showing just how effective (or not!) these methods are in real time.

Graham Ellis : Six examples in computational group cohomology

Six short examples of machine computations in group cohomology will be presented using the GAP system for computational algebra. Each short example will be preceded by a longer explanation of the underlying theory. The examples will span:

(1) group theory and low-dimensional cohomology;

(2) linear algebra and mod-p cohomology of a finite group;

(3) combinatorial topology and the integral cohomology ring of a fundamental group;

(4) contracting homotopies and the low dimensions of a classifying space of a finite group;

(5) homological perturbation theory;

(6) integral cohomology of Bianchi groups and an approach to Hecke operators.

Further examples can be found at :

`https://gap-packages.github.io/hap/tutorial/chap0_mj.html`.

2 Talks

Amina Abdurrahman : TBA

Alex Bartel : Vignéras pairs of isospectral manifolds

In 1966 Kac asked the famous question "Can one hear the shape of a drum", in other words, what geometric properties of a Riemannian manifold are determined by the spectrum of the Laplace operator, acting on the space of functions on the manifold, or more generally on the spaces of differential i-forms for all i. After a few ad-hoc examples of pairs of non-isometric but isospectral manifolds had been found, in 1980 Vignéras proposed a general construction that, under some additional conditions, yields isospectral manifolds, using non-conjugate orders in quaternion algebras over a number field. Those conditions are still not fully understood, and all hitherto known general criteria yielded much stronger relationships between those manifolds than mere isospectrality. In joint work with Aurel Page, we revisit Vignéras's construction, give fine criteria for various isospectrality relationships, and hence produce quite "exotic" pairs of isospectral manifolds.

Naomi Bredon : On ADEG-polyhedra in hyperbolic spaces

In this talk, we discuss the classification of hyperbolic Coxeter polyhedra in dimensions beyond 3. After a brief overview of known classification results, we present a bound for the dihedral angles of Coxeter polyhedra with mutually intersecting facets, together with a recipe to classify all such polyhedra. We provide a new Coxeter polyhedron in dimension 9 and complete the classification of ADEG-polyhedra.

Michelle Chu : Surfaces in two-bridge knots and components in the character variety

The $SL(2,\mathbb{C})$-character variety of the fundamental group of a hyperbolic 3-manifold encodes a lot of topological and geometric information about the manifold. As an algebraic set, this character variety can have multiple algebraic components. In this talk I will mention both old and new results on when characters at the intersection of distinct components in the character variety of two-bridge knot groups produce splittings of the fundamental group over surface subgroups.

Martin Deraux : Smooth complex hyperbolic surfaces with a single cusp

Apart from very low dimensions, it is difficult to construct 1-cusped locally symmetric manifolds. The first example of a 1-cusped real hyperbolic 4-manifold was constructed by Kolpakov-Martelli. For the 2-dimensional complex hyperbolic case, examples have been known for a while that almost did the job (orbifolds with one cusp, or smooth surfaces with a small number of cusps). I will explain recent joint work with Stover, where we construct an infinite tower of

1-cusped complex hyperbolic surfaces.

Sami Douba : On Vinberg's notion of quasi-arithmeticity

In the 60s, Vinberg introduced his notion of quasi-arithmeticity, a weakening of the notion of arithmeticity for lattices in semisimple groups. It follows from the superrigidity theorems of Margulis and Gromov–Schoen, and from work of Esnault–Groechenig, that the existence of quasi-arithmetic lattices that are not in fact arithmetic is essentially a real hyperbolic phenomenon. I will explain how quasi-arithmeticity arises naturally in the study of arithmetic real hyperbolic lattices themselves, and is indeed a useful tool for distinguishing commensurability classes of non-(quasi-)arithmetic lattices. Time permitting, I will also discuss some open questions. This talk is partially based on joint work with Nikolay Bogachev and Jean Raimbault.

Mikołaj Frączyk : Minimal submanifolds of locally symmetric spaces and low dimensional actions

A group G has Serre's property FA if every action of G on a tree has a global fixed point. It is well known that groups with property (T) have property FA. There is a natural generalization of propery FA to higher dimensional actions on simplicial complexes, called property FA_{r} where one requires that any action on an r-dimensional CAT(0) simplicial complex has a global fixed point. Farb proved that $\mathrm{SL}_n(\mathbb{Z})$ enjoys property $\mathrm{FA}_{\mathrm{n-1}}$, but it remains elusive for other higher rank lattices. In a joint work with Ben Lowe, we develop a new method for establishing property FA_{r}, using minimal submanifolds and min-max theory. As a consequence we show that any cocompact lattice in $\mathrm{SL}_n(\mathbb{R})$ has property $\mathrm{FA}_{[\mathrm{n}/8]\text{-}1}$ and lattices in the exceptional rank one group $F_4^{(-20)}$ have property FA_2.

Claudius Kamp : Plancherel Convergence and Zeta Functions

In this talk we will discuss a certain kind of convergence for hyperbolic surfaces, called Plancherel convergence, and describe consequences for the convergence of the Selberg zeta functions of the involved surfaces. In particular, we will focus on constraints on the spectral geometry of the hyperbolic surfaces posed by Plancherel convergence.

Arielle Leitner : The Chabauty Space of Subgroups of PSL(2,R)

We will discuss the topology on the space of subgroups of $\mathrm{PSL}(2,\mathbb{R})$ with the Chabauty topology. We will take a tour of elementary subgroups and lattices, and use grafting to parametrize deformations of the latter. We will describe some connected components of $\mathrm{Sub}(\mathrm{PSL}(2,\mathbb{R}))$, and their closures. Joint work with Ian Biringer and Nir Lazarovich

Michael Lipnowski : Sorting on manifolds

Aurel and I devised an algorithm to cover large compact (congruence arithmetic) locally symmetric manifolds M by balls of size 1 in time $\sim \mathrm{vol}(M)^2$. The exponent 2 in our approach arises for similar reasons that the exponent 2 arises in naive algorithms for sorting N real numbers in time $\sim N^2$. In this talk, I'll report on the thesis of my Master's student Asa Kohn, which pursues an analogue of "insertion sort on manifolds" with the dream of improving the exponent 2 to exponent $1+\epsilon$ as in classical insertion sort.

Plinio Murillo : On kissing number of hyperbolic manifolds

Motivated by a result of M. F. Bourque and B. Petri, we constructed a sequence of hyperbolic manifolds with a large number of closed geodesics of shortest length. The aim of this talk is to explain what "large" means, and how arithmetic groups enter in this context. This will involve results in collaboration with Cayo Dória and Emanoel Freire.

Pierre Py : Finiteness properties and complex hyperbolic lattices

The notion of being finitely generated or finitely presented for a group G can be given a geometric interpretation: for instance a group G is finitely presented if and only if it is the fundamental group of a finite CW-complex of dimension 2. This interpretation leads to higher-dimensional generalizations of the notion of finite generation/presentation (being of type $\mathrm{F_n}$ or of type F). These higher dimensional properties are known as finiteness properties and were introduced by Wall. In this talk I will recall the definition of these properties and will explain how to describe the finiteness properties of the kernels of certain homomorphisms from cocompact arithmetic lattices in $\mathrm{PU}(n,1)$ to $\mathbb{Z}$ or $\mathbb{Z}^2$. This is based on a joint work with Claudio Llosa Isenrich.

James Rickards : Totally geodesic surfaces in Bianchi orbifolds

Does there exist an embedded closed totally geodesic surface in a Bianchi orbifold that is separating? I will describe a joint project (with Junehyuk Jung and Sam Kim), where we are computationally studying this question. Background material of geodesics and fundamental domains in arithmetic Fuchsian groups will be introduced, transitioning into the higher dimensional Bianchi orbifolds. Algorithms used to study the main question will be described, and a few conjectures borne from the data will be mentioned.

Rafael Sayous : Asymptotic gaps in the Farey fractions of an imaginary quadratic number field

We will quickly present a well-known result on the gaps of real Farey fractions, namely the asymptotic density of R. R. Hall (1970) obtained with number theoretic arguments, then found again by others using arguments from dynamics, in particular by J. Marklof (2013) using homogeneous dynamics on $\mathrm{SL}_2(\mathbb{R})/\mathrm{SL}_2(\mathbb{Z})$. Let K be a quadratic imaginary field and denote by O_K its ring of integers. After adapting the method of Marklof and lifting a joint equidistribution result of J. Parkkonen and F. Paulin (2022), we will obtain a result on the asymptotic distribution of gaps in the fractions of elements of O_K. The homogeneous dynamic we study will be set on $M\backslash\mathrm{PSL}_2(\mathbb{C})/\mathrm{PSL}_2(O_K)$ (with M the maximal diagonal compact subgroup). We will prove the existence of an asymptotic density for these gaps, the cumulative distribution function will be described geometrically, and from this description we will be able to estimate its tail distribution in the case of Gaussian or Eisenstein fractions.

Suzanne Schlich : Primitive-stable and Bowditch representations

In this talk, we will introduce Bowditch representations of the free group of rank two (defined by Bowditch in 1998) along with primitive-stable representations (defined by Minsky in 2010) acting on Gromov hyperbolic spaces. Minsky first introduced primitive-stable representations in PSL(2,C) to construct an open domain of discontinuity in the character variety strictly containing the set of convex-cocompact representations. We will discuss the equivalence between primitive-stable and Bowditch representations. We will also introduced simple-stable representations of a surface group and give a similar result in the case of the four-punctured sphere. Then we will see a higher rank analogue of the simple-stable property and that we can certify this property (this is joint work with Max Riestenberg).

Matthew Stover : Products of curves as ball quotients

For any $g_1, g_2 \geq 0$, I will describe explicit constructions of ball quotients birational to a product $C_1 \times C_2$ of smooth projective curves C_j of genus g_j. The only prior examples were $g_1 = g_2 = 0$, due to Deligne–Mostow and rediscovered by many others, and a lesser-known product of elliptic curves whose existence follows from work of Hirzebruch. Combined with related new examples, this answers the rational variant of a question of Gromov in certain cases. It also follows that every simply connected 4-manifold is dominated by a complex hyperbolic manifold. All examples are arithmetic, and even arithmeticity of Hirzebruch's example appears to be new.

Anne-Edgar Wilke : Kempf-Ness covariant and reduction theories for actions of arithmetic groups

Let G be a reductive algebraic group acting on an algebraic variety X, where G and X are both defined on a number field k. Let Γ be an arithmetic subgroup of $G(k)$. The problem considered in this talk is to construct a fundamental domain for the action of Γ on $X(k)$. The points of this domain are then said to be *reduced*. Ideally, one also wants an efficient algorithm to compute a reduced point representing any given point of $X(k)$. I will show an approach which answers these questions in a very general setting, relying on a geometric invariant theoretic object: the Kempf-Ness covariant.

David Xu : Irreducible actions of Coxeter groups on the infinite-dimensional hyperbolic space

Similarly to Euclidean spaces, there is an infinite-dimensional analog for the hyperbolic spaces. This space enjoys all the "geometric" properties of its finite-dimensional siblings. However, the topological aspects are more involved as in finite dimension. In particular, the notion of discrete isometry groups has to be specified in this context. In this talk, I will present the infinite-dimensional hyperbolic space and describe some of its properties. Then, I will discuss about how one can construct Coxeter groups acting irreducibly on it.